Mathematical Analysis of Fluid Dynamics

We study the stability and time-asymptotic behaviour of solutions to viscous fluid-dynamical equations.
The domain the fluid occupies is unknown in free-boundary problems, and many studies have been done under various physical-boundary conditions, such as 2-fluid flow and free-surface problems. Viscous fluids with fixed boundaries are usually governed by the nonlinear dissipative system. If the regularities and the time-asymptotic behaviour of the free boundary are also taken into account, we need another approach whereby the system has properties that are not only dissipative, but hyperbolic. We employ the dissipative structure and try to determine the hyperbolic properties of the solutions.
Fluid-dynamical equations can be observed in describing various physical, life and social phenomena. We expect our analysis to be applied to the analysis and explanation of natural phenomena.

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p - \nu \Delta u = 0 \qquad x \in \mathbb{R}^3,\ t > 0$$

$$\nabla \cdot u = 0 \qquad x \in \mathbb{R}^3,\ t > 0$$

The Navier–Stokes equations are one famous system describing the motion of a viscous fluid

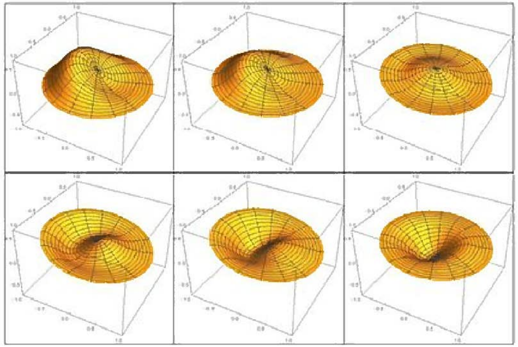
About Researcher

HATAYA Yasushi, Ph.D.

Ph.D., 2004, Kyoto University

Partial Differential Equations

Prof. Hirosawa's current research interest is in initial-value problems for partial differential equations of the hyperbolic type. He is particularly focused on deriving the asymptotic behaviour of the energy and smoothness properties of the solutions for linear and nonlinear wave equations whilst taking account of the singular behaviours of variable coefficients and nonlinearities. Although the problems he addresses are purely mathematical, they can be reduced to problems of mathematical physics. Indeed, wave equations describe actual physical phenomena, and variable coefficients denote non-constant external forces and non-homogeneous structures of the medium. He has observed many complicated wave properties as well as interesting problems from the mathematical point of view that correspond to actual physical phenomenon such as resonance, blow-up, decay and scattering. Some of these properties cannot be easily considered in numerical analysis; however, Prof. Hirosawa's recent proposal was demonstrated to be applicable for estimating the qualitative and quantitative properties of the solutions to such problems.



Time variation of a solution to the wave equation on a unit disc with Neumann boundary condition

About Researcher

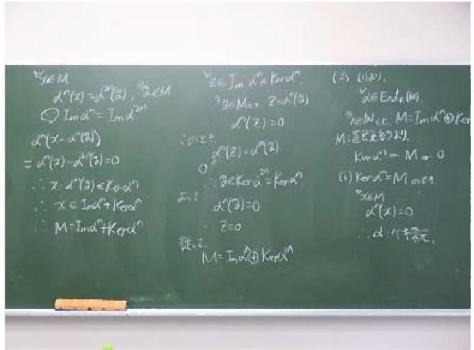
HIROSAWA Fumihiko, Ph.D.

Ph.D., 1998, University of Tsukuba

WEB ≫ http://ds0n.cc.yamaguchi-u.ac.jp/~hirosawa/index.htm

Structure of Rings

Various artinian rings, e.g. quasi-Frobenius, serial or Harada rings, are based on local quasi-Frobenius rings. We studied the construction of local quasi-Frobenius rings with Jacobson-radical-cubed zeros. Moreover, we studied local artinian rings of the split type, in particular, the structure of locally quasi-Frobenius rings of split type, and gave a method for constructing these rings. We study such (not necessarily commutative or artinian) rings and constructions of rings as described above. Using these constructions, we present various examples of typical or interesting rings.



In our seminar

About Researcher

KIKUMASA Isao, Ph.D.

Ph.D., 1990, Okayama University

Sums of Arithmetical Functions

I investigate the asymptotic expansions of sums of arithmetical function and study the analytical properties of zeta and double-zeta-functions with number-theoretical coefficients of certain arithmetical functions. My main research areas are as follows:

(1) Analytical properties of Ramanujan sums, Anderson-Apostol sums and others, as well as their applications.

(2) Asymptotic properties of the Euler function, the Dedekind function, the divisor function, the gcd-sum function, the lcm-sum function and others, as well as their applications.

(3) Analytical and asymptotic properties of double-zeta-functions of the Euler-Zagier type, as well as their applications.

(4) Analytical properties of exponential sums and exponential integrals, as well as their applications.

Our research concept and related devices

About Researcher

KIUCHI Isao, Ph.D.

Ph.D., 1992, Keio University

Lifting Modules, Extending Modules and their Generalisations

Injective and projective modules are the most important and have thus become the central research objects of ring and module theory since the mid-19th century. An injective module has a significant property called the extending property, and an extending module is defined by this property. On the other hand, a lifting module is defined by the dual concept. The importance of extending modules and lifting modules in ring and module theory became obvious in 1980s.

Our main research objects are these modules and generalisations. In particular, we recently have studied the following questions:

(1) Is there a ring whose lifting right R-modules all have indecomposable decompositions? As subproblem, does a lifting module over a semiperfect ring have an indecomposable decomposition?

(2) Under what circumstances is a direct sum of lifting (resp. extending) modules also lifting (resp. extending)?

$$\begin{array}{ccccccccc}
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
A_1 & \oplus & A_2 & = & & & A & & \\
h_1 \downarrow & & h_2 \uparrow & & & & f \downarrow & & \\
B_1 & \oplus & B_2 & = & B & \underset{g}{\rightarrow} & X & \rightarrow & 0
\end{array}$$

$$A \oplus B = \underset{\ker(f-g)}{\underset{\bigcap}{\underline{\langle A_1 \xrightarrow{h_1} B_1 \rangle \oplus \langle B_2 \xrightarrow{h_2} A_2 \rangle}}} \oplus A_2 \oplus B_1$$

This diagram appears in a study of the direct sums of lifting modules

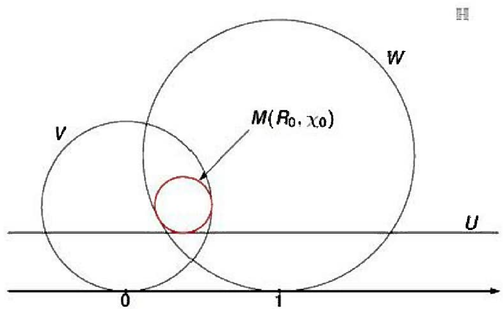
About Researcher

KURATOMI Yosuke, Ph.D.

Ph.D., 2003, Yamaguchi University

Theory of Riemann Surfaces

A Riemann surface is, by definition, a 1-dimensional connected complex manifold. The Riemann sphere and its subdomains are trivial examples of Riemann surfaces. By the general uniformisation theorem, every Riemann surface of genus zero is conformally embedded into the Riemann sphere. Thus, functional theory on such Riemann surfaces is essentially a special case of functional theory on plane domains. In other words, the core of the theory of Riemann surfaces should be occupied by Riemann surfaces of positive genus, that is, those with handles. We are interested in the existence of holomorphic mappings of one Riemann surface onto another. Small topological or analytical conditions on holomorphic mappings could result in strong restrictions. For example, let R and S be doubly connected Riemann surfaces. If the modulus of S is less than that of R, then all holomorphic mappings of R onto S are homotopic to constant mappings. We have been systematically studying such phenomena.



Extremal lengths of three homology classes determine the moduli disc.

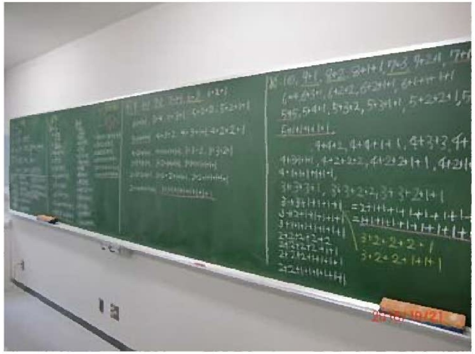
About Researcher

MASUMOTO Makoto, Ph.D.

Ph.D., 1988, Kyoto University

Analytic Number Theory

When I was a child, I visited many mandirs, especially Rokkaku-do temple with my grandfathers and grandmothers. There, I learned some Sanskrit phrases. I admired India and hoped to go and study Indian philosophy, for example, the Upanishads. Long afterward, when I was an undergraduate student, I read Tagore's Gitanjali and I became familiar with a great Indian mathematician by the name of Srinivasa Ramanujan. Needless to say, I could not understand his theory, but I was captivated by his formulas, especially (¥frac{1}{4}¥cot¥frac{¥theta}{2}+¥sum_{n=1}^{¥infty} ¥frac{q^n}{1-q^n} ¥sin n¥theta)^2=(¥frac{1}{4} ¥cot ¥frac{ ¥theta}{2})^2 + ¥sum_{n=1}^{¥infty} ¥frac{q^n}{(1-q^n)^2} ¥cos n¥theta +¥frac{1}{2} ¥sum_{n=1}^{¥infty} ¥frac{nq^n}{1-q^n}(1- ¥cos n ¥theta) (in `On certain arithmetical functions', Transactions of the Cambridge Philosophical Society, XXII, 9, 1916, 159--184). I thus decided to study analytic number theory. When I was a graduate student, I visited Harish-Chandra Research Institute in Allahabad, India. I had a very happy time there as I saw Ganga every day. Indian food is very delicious. Some boy read the book Mahabharata to me at the institute. Now remembering India, I study analytical number theory.



Our research concept and related devices

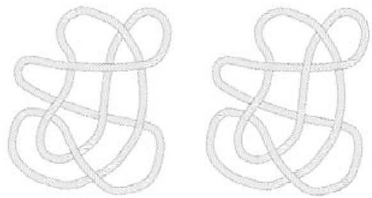
About Researcher

MINAMIDE Makoto, Ph.D.

Ph.D., 2009, NAGOYA UNIVERSITY

Study on Invariants for Knots and Links

My research field is knot theory in topology and a major subject of interest is polynomial invariants for knots and links. A link with m components is a subset of the 3-sphere S^3 consisting of m disjoint simple closed curves. A link of one component is called a knot. Two links, L and L', are equivalent if there is an orientation-preserving auto-homeomorphism h of S^3 such that h(L) = L'. The main purpose of knot theory is to classify knots and links. An effective method for this classification is to make use of invariants, in particular, polynomial invariants. In this sense, polynomial invariants play an important role in various situations in the study of knot theory. Hence, it is significant to discover a polynomial invariant and characterise its properties. There are two knots in the figure on the right-hand side. One is the trivial knot, which is equivalent to a simple closed curve on a plane in S^3, and the other is a non-trivial knot. These can be distinguished by a variety of invariants, for example, the genus, the signature, the Alexander polynomial and the Jones polynomial.



Examples of knots: Which is the trivial knot?

About Researcher

MIYAZAWA Yasuyuki, Ph.D.

Ph.D., 1993, Osaka City University

Variational Problems on Riemannian Manifolds

Our research is on variational problems on Riemannian manifolds. Variational problems constitute an important field in mathematics. In this field, we minimise a given quantity and investigate the properties of minimisers, together with their applications; for example, objects move on shortest paths, i.e. length-minimising paths, and soap films and bubbles choose area-minimising surfaces by surface tension. There is a fundamental principle that states that all phenomena in the world can be characterised by solutions of variational problems. I believe also that any concept in the sciences can be described from the viewpoint of variational problems and related fields. Recently, we have been interested in mappings between Riemannian manifolds, which are known as symphonic mappings and C-stationary mappings. These mappings come from the concept of conformality, which is important in both mathematics and physics.

I produced four texts on differential geometry of curves and surfaces and mathematical logic, which contained humour and mathematical maxims.

About Researcher

NAKAUCHI Nobumitsu, Ph.D.

Ph.D., 1995, Osaka University

Research on the Hilbert Functions of Local Rings

In my laboratory, we are working on commutative algebra, which was started by D. Hilbert, E. Noether, and other mathematicians, at the beginning of 20^{th} century. It is one branch of algebra that explores the structure of commutative rings.
We are particularly interested in the theory of Hilbert functions of commutative Noetherian local rings. The notion of the Hilbert function plays an important role in commutative algebra. Starting from the pioneering work of G. D. Northcott and J. D. Sally in the middle of the 20^{th} century, many researchers have been engaged in the development of the theory. The purpose of my research is to characterise local rings and their ideals in terms of Hilbert functions.

Key words and phrases of our research are stated as follows:
Commutative rings, Noetherian local rings, primary ideal, Hilbert function, Hilbert coefficient, multiplicity, Rees algebra, associated graded ring, fibre cone, Cohen-Macaulay local rings, Buchsbaum local rings, Sally module

This photo shows a seminar of by a master's student.

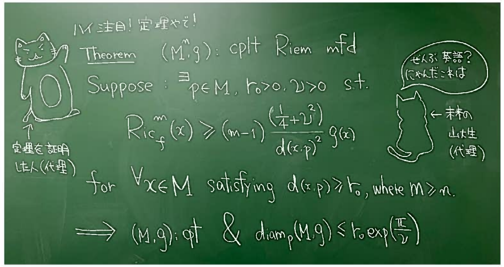
About Researcher

OZEKI Kazuho, Ph.D.

Ph.D., 2009, Meiji University

Geometric Analysis on Riemannian Manifolds

My research is on a field of geometry called differential geometry. Roughly speaking, differential geometry is a study that is deeply related to the question of "what is the shape of our universe", and makes full use of calculus and linear algebra to investigate "manifolds". Here, a "manifold" is a generalization of a curve and a surface into a high-dimensional concept. A curve is a one-dimensional manifold and a surface is a two-dimensional manifold. Among manifolds, one object I am researching in particular is called a "**Riemannian manifold**". A Riemannian manifold is a manifold equipped with a measure of the length between two arbitrary points. Euclidean spaces and spheres are examples of Riemannian manifolds. Among Riemannian manifolds, I am interested in an "**Einstein manifold**" and a "**Ricci soliton**", which are closely related to Einstein's theory of relativity. I am investigating the behavior of the diameters and volumes of such Riemannian manifolds.



A theorem exhibiting a sufficient condition for a Riemannian manifold to have a finite diameter.

About Researcher

TADANO Homare, Ph.D.

Ph.D., 2015, Osaka University

Representation Theory of Finite Dimensional Algebras

My research field is the **representation theory** of finite dimensional algebras. The aim is to study finite dimensional algebras by analyzing the structures of module categories. In particular, I am interested in (strongly) **quasi-hereditary algebras** which are a special class of finite dimensional algebras. Quasi-hereditary algebras were introduced by Cline, Parshall and Scott to study highest weight categories which arose in the representation theory of semisimple complex Lie algebras and algebraic groups. Motivated by Iyama's finiteness theorem of representation dimensions of algebras, Ringel introduced the notion of strongly quasi-hereditary algebras from the viewpoint of highest weight categories. It is known that strongly quasi-hereditary algebras have a better upper bound of global dimension than that of general quasi-hereditary algebras. I gave a characterization of strongly quasi-hereditary algebras in terms of **rejective chains**. The purpose of my research is to give a construction of algebras with finite global dimension by using rejective chains.

At the 50th Symposium on Ring Theory and Representation Theory

About Researcher

TSUKAMOTO Mayu, Ph.D.

Ph.D., 2018, Osaka City University